高次精度，衝撃波捕獲，磁場のソレノイダル条件を自動的に満たす保存型の新しい圧縮性 MHD 計算スキーム

河合 宗司 [1]
[1] JAXA 宇宙研

High-order accurate, shock-capturing, and divergence-free-preserving conservative scheme for compressible magnetohydrodynamics

Soshi Kawai[1]
[1] ISAS/JAXA

http://flab.eng.isas.jaxa.jp/member/kawai/

Understanding magnetohydrodynamics (MHD) in compressible flows involving turbulence, shock waves, and their interactions is of significant interest in a wide range of both scientific and engineering applications, such as the magnetosphere, heliosphere, and astrophysics. A significant challenge in the field of simulating the compressible MHD flows is to establish a numerical scheme that is able to accurately simulate flows involving turbulence, shock waves and their interactions while satisfying divergence-free magnetic fields. The numerical algorithm needs to satisfy three requirements simultaneously: the scheme needs to be high-order accurate to accurately resolve the broadband scales of turbulence, robustly capture shock waves, and also automatically preserve the divergence-free property. To our knowledge, however, there is no single numerical method that satisfies the three three requirements.

In this study, we propose a new and very simple conservative numerical scheme for solving compressible MHD equations. The proposed method simultaneously satisfies the three requirements: high-order accurate, shock-capturing, and automatic divergence-free preservation, while incorporating the strong conservative form of governing equations and co-located solver. The analysis of the numerical method in compressible MHD flows directly leads to the proposed simple yet effective physically-consistent numerical diffusion terms that maintain the three requirements. The proposed method dynamically adds non-linear physically-consistent artificial diffusivity locally in space to capture shock waves while satisfying the divergence-free magnetic fields, and a high-order central differencing scheme (sixth-order compact differencing scheme in this study) resolves a broad range of scales in flows. The method is simple, low computational cost, and ease of implementation. The capability of the proposed method is verified through several one- and two-dimensional compressible MHD flow problems with shock waves.

磁気圏，太陽圏，電離圏，惑星圏および天体現象を含め宇宙プラズマ環境に起因する様々な流体物理現象には，乱流現象や乱流と衝撃波との干渉がキーとなる事例も少なくない．この様な流体現象に対して，圧縮性 MHD 方程式を数値的にシミュレートし，複雑な流体物理を調べるためには，ブロードバンドなスペクトルを持つ乱流現象を精度良く解析できる中心差分系の高次精度スキームを用いつつ，衝撃波をロバストに捕獲し，かつ磁場のソレノイダル条件を満足させる必要がある．しかし現状，著者の知る限り，これらの高次精度，衝撃波捕獲，磁場のソレノイダル条件を自動的に満たす計算スキームは存在しない．

そこで本研究では，高次精度，衝撃波捕獲，磁場のソレノイダル条件を自動的に満たす保存型の新しい圧縮性 MHD 計算スキームを提案する．まず一般的な形で，高次精度，衝撃波捕獲，磁場のソレノイダル条件を自動的に満たすスキームの条件を解析的に導出する．ここでのポイントは，提案する計算スキームが数学的に行われる解析的な導出結果に基本的に基づくが，一方で支配方程式に本来備わっている圧縮性 MHD の流体物理を乱さないよう，物理的にコンシステントな局所人口粘性項を構築する点にある．すなわち本手法は数値計算の数学的な側面と圧縮性 MHD 方程式の物理的な側面の両側面を満たす計算法であるとも言える．提案する手法は，高次精度中心差分法（本研究では高解像度な 6 次精度コンパクト差分法を用いた）をベースとし局所人口粘性を付加する手法で，コロケート格子を用いる非常にシンプルなもので，計算コストも低く，導入も非常に容易である．幾つかの 1 次元，2 次元問題を通して，本手法の有効性を実証する．