HLL Riemann Solver with Divergence-free Constraint for Two-Fluid Simulations

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There has been growing interest in incorporating small scale physics in Magnetohydrodynamic (MHD) simulations, because rapidly increasing computational resources may enable one to use sufficiently small grid size even in macroscopic simulation models, such as global MHD simulation of a planetary magnetosphere. For instance, attempts have been made to include Hall physics in global magnetospheric simulations. The Hall term introduces dispersion in the eigenmode of the system, making numerical treatment much more difficult due to the appearance of high-frequency whistler waves. Since the phase speed of whistler waves increases without bound in the Hall-MHD, fully or partially implicit treatments are sometimes employed for numerical stability.

In this report, we extend the conventional Hall-MHD to two-fluid equations with charge neutrality constraint, that takes into account finite electron inertia effect. Because of the assumption of charge neutrality, the number of eigenmodes in this system is the same as the ideal MHD, whereas dispersive characteristics appear due to ion and electron inertia effects. In contrast to the Hall-MHD, there is a limit in the maximum phase speed introduced by finite electron inertia, a property desirable for numerical simulations.

The system may be written in a conservative form that resembles the MHD, with modification in the generalized Ohm's law. Therefore, the mass, momentum, and energy conservation laws may exactly be satisfied by using a proper conservative scheme. We have implemented the single state HLL Riemann solver for this system, that is combined with Upstream Constrained Transport (UCT) scheme of Londrillo & Del Zanna (2004). The UCT scheme employing the staggered definition of magnetic field is specifically designed for the divergence-free constraint and can be combined with any Riemann solvers. The present scheme thus guarantees the divergence-free condition for the magnetic field. Several numerical examples have shown that the code is able to capture discontinuities as well as dispersive signature arising from finite electron and ion inertia. The code, now being extended to incorporate kinetic effects, may become a useful next generation simulation model with a lot of applications in space physics and astrophysics.