HLLI-UCT法：HLLI近似Riemann解法と風上型CT法による誘導方程式の高解像度化

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HLLI-UCT: improving resolution of induction equation by HLLI approximate Riemann solver and upwind CT method

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The induction equation is one of the basic equations in the magnetohydrodynamics (MHD). Since this equation has the form of curl rather than divergence of its flux, not like in other conservation equations, the divergence of the magnetic field must be preserved, and it should be zero due to nonexistence of a magnetic monopole. Any numerical scheme which breaks this constraint leads to spurious force along the local magnetic field and returns a physically incorrect solution.

A constrained transport (CT) algorithm is a feasible prescription to construct a divergence-free scheme by the use of staggered arrangement for the magnetic and the electric fields, which guarantees that the divergence error is kept within machine precision. As a CT method simply requires the electric field evaluated at cell edges, there has been several variants of the CT method depending on interpolation schemes for the electric field. An HLL-upwind CT (UCT) method is one of the most important schemes, which enables one to construct a higher-order scheme by combination of an arbitrary interpolation method and an HLL approximate Riemann solver. As long as the scheme involves simple HLL averaging, however, one inevitably suffers from the dissipative nature of the HLL Riemann solver, especially for high-beta plasmas. Recently, moreover, it has become clear that the resolution of the HLL-UCT method cannot be improved simply by replacing the HLL scheme with other HLL-type, non-linear, and high-resolution Riemann solvers such as HLLC, HLLR and HLLD. The problem arises from the fact that these non-linear Riemann solvers cannot be separated into an averaging term and a dissipation term in contrast to the HLL scheme. This fact leads the effect of the numerical dissipation to enter into the interpolated electric field twice, which is the so-called dissipation doubling.

In this work, we made use of the idea that the dissipation doubling can be avoided by employing the recently proposed HLLI approximate Riemann solver, where substructure of a Riemann fan is linearly reconstructed consistently with the eigensystem. By combining the HLLI method and the upwind CT technique, we developed an HLLI-UCT scheme. The use of our scheme successfully overcomes the dissipative property of the original HLL-UCT scheme. Moreover, because the HLLI scheme does not rely on specific non-linear jump conditions, which are approximated in a heuristic manner in the case of a non-linear HLL-type Riemann solver, it is also possible to apply the HLLI-UCT to other non-ideal MHD systems if the eigensystem can be obtained; of course it can be obtained for any hyperbolic system. Examples include the kinetic MHD system with pressure anisotropy, which may play an important role in collisionless plasmas. Even in such a system for which a non-linear Riemann solver is not developed, one can apply the HLL-UCT method straightforwardly. In our presentation, we will discuss the impact and some technical details of the new HLLI-UCT scheme.